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Wage bargaining and adjustment to shocks

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Abstract

There has been a marked difference in the way developed economies have responded to the 'shocks' experienced in the past decade in terms of their fluctuations in employment, wages and output. It is sometimes argued that these differences reflect different institutional arrangements for wage setting. Yet the theory of implicit contracts shows that, under symmetric information, employment and output levels would be the same whether there were competitive spot labour markets, *ex post* bargaining, or *ex ante* bargaining (provided such bargains included employment). For these differences in wage setting to matter, asymmetries in information are usually introduced. In this chapter we explore a different reason why these different wage setting environments would matter, namely that firms can use both wage contracts and 'real smoothing' to protect workers against shocks. We investigate how different wage setting institutions will induce too much or too little real smoothing (relative to the efficient level), and hence induce different fluctuations in employment or output in response to the same shocks.

1. Introduction

A topic of recent research and much debate is the different performance of OECD countries in response to the various shocks of the past 10 to 15 years (exchange rate volatility, oil price increases, competition from NICs, etc.)—for example, Bruno and Sachs (1985). While part of these differences can be accounted for by differences in the policies pursued by governments, significant differences between countries remain to

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be explained, and it is often argued that differences in institutional structures are important, particularly with respect to the impact of institutional structure on the wage setting procedures (Bean, Layard and Nickell 1986, for example).

Yet, when one turns to the theoretical literature for reasons why different wage bargaining structures should affect the way economies respond to shocks, there is not always support for this view. The literature we have in mind is that on implicit labour contracts where the issue of how workers might seek protection from shocks is addressed explicitly (see Hart 1983; Azariadis and Stiglitz 1983; Hart and Holmstrom 1987; Rosen 1985, for surveys). As is now well known, in the original formulation of these contract models due to Azariadis (1975), Bailey (1974) and Gordon (1974), where it was assumed that both parties could observe the state of the world (symmetric information) the theory could explain why wages would not fluctuate as much as income over the cycle, but it could not explain why there would be excessive fluctuation in employment. More precisely, under symmetric information, *ex ante* efficient contracts would also be *ex post* efficient, so that the level of employment would always be efficient, and under a widely adopted assumption that the marginal disutility of leisure is constant, the efficient level of employment would be the same as would prevail in Walrasian spot labour markets. Thus it makes no difference whether the wage setting process is represented as taking place under spot labour markets, *ex post* bargaining, or *ex ante* bargaining, the level of employment and hence output will be the same for each state of the world, though of course the share of wages in income would be quite different.

One way to escape from this conclusion is to introduce asymmetric information (Azariadis 1982; Grossman and Hart 1981, 1982; Chari 1982; Green and Kahn 1982), say by assuming that firms can observe the shock, but not workers. Then in many states of the world employment under *ex ante* wage bargaining will be inefficient, and while different assumptions about workers' utility can lead to either underemployment in bad states or overemployment in good states, in either case there will now be greater fluctuations in employment and output with *ex ante* wage bargaining than with *ex post* bargaining (though under the simplifying assumption of constant marginal utility of leisure, *ex post* bargaining will still be equivalent to spot labour markets). A difficulty with this approach, though, is that if one is dealing with macroeconomic shocks it is perhaps implausible to assume that information is asymmetric, for even if the effects of the shock on different firms may differ, as long as shocks are correlated, then observing macro variables like aggregate employment will be equivalent to observing the state of the world.

In this paper we retain the assumption of symmetric information, and seek to provide an alternative way of having the wage setting process affect the response of the economy to shocks. This rests on the observation that firms can protect workers from shocks more directly, for example, through stock building, choice of capital stock, or choice of production structure in terms of the degree of *ex post* substitutability factors. We shall refer to this as *real smoothing*, and suppose that the amount of real smoothing is also bargained over by workers and firms. As we shall show, efficient outcomes require that firms and workers bargain jointly over the amount of real smoothing and the level of wage and employment in each state of the world. Different forms of wage setting will then induce differences in the level of real smoothing chosen, and so affect the way the economy can respond to real shocks.

We shall examine two particular forms of inefficiency in the wage setting process. First, we shall assume that wages and employment are bargained *ex post* rather than *ex ante*. Second, we assume that while wages and employment are bargained *ex ante*, the bargaining strength of the firm is different in the bargain over wages and employment than in the bargain over the level of real smoothing. This difference in bargaining strength is simply a device, following Manning (1985), for capturing the kind of imperfection discussed by Grout (1984), Crawford (1988), Ulph (1989), that arises from the inability of firms and workers to sign long-term contracts. So we could imagine firms and workers being able to sign (contingent) wage and employment contracts for a medium-term period (say 2 to 3 years) but not able to commit themselves to longer-term contracts that would be required to tie wage and employment decisions in with decisions on levels of capital investment or choice of production structures. As in the case of contracts under asymmetric information, we cannot unambiguously sign the effects of these inefficiencies on the level of real smoothing, though for a broad range of parameter values there is a presumption that the first source of inefficiency will lead to too much real smoothing, while the second will lead to too little.

We have chosen to develop our argument in the context of about the simplest model we could devise. To a large extent the rather special assumptions were chosen to bring our model as close as possible to the special case employed in the implicit contract literature whereby, for any given level of real smoothing, different wage setting procedures (under symmetric information) would have no effect on employment or output. Specifically, we consider a single firm dealing with a single union whose members could all be employed outside the firm at a given reservation wage, and suppose that the union distributes income equally among all its members. These assumptions ensure that the union is concerned only about how much its members earn in excess of reservation wages, and is not concerned directly about employment which can be taken to be chosen to be *ex post* efficient, i.e. workers are employed in the firm until their marginal revenue product equals the reservation wage. Differences in the wage setting procedure, reflected in the two distortions from full efficiency noted above, have their effect only through the impact on the level of real smoothing. The two other simplifications are the use of only two states of the world and particular forms for the workers' and firms' utility functions. Both are chosen for tractability, and as Hart and Holmstrom (1987) note, having more than two states of the world rarely yields any additional insights.

The structure of the paper is as follows. In §2 we set out the model and summarize the outcome of wage (and employment) bargaining under *ex ante* and *ex post* bargaining for a given level of real smoothing. In §3 we examine the determination of real smoothing, beginning with the efficient outcome, and then examine the effect of the two distortions in bargaining already referred to. Since the results will be ambiguous analytically, in §4 we present some numerical simulations to demonstrate the orders of magnitude of the effect of the distortions, and offer some conclusions in §5.

2. The model and the wage bargaining outcome

We consider a single firm bargaining with a single union whose membership is large enough to provide all the workers the firm is ever likely to need. There are two states of the world, 1 and 2, with 1 the bad state, which occur with probabilities ρ and $1-\rho$ respectively, assumed to be common knowledge. In the absence of any smoothing the surplus accruing to the firm in the two states would be $\underline{\pi}$, $\overline{\pi}$ respectively, with $0 \leq \underline{\pi} < \overline{\pi}$, where surplus denotes revenue less costs of all factors of production, including labour valued at the reservation wage. We shall denote *real smoothing* in an abstract form as the sacrifice of surplus, z, in the good state of the world in order to increase surplus in the bad state by an amount $f(z)$, so that smoothed surpluses are

$$\pi_1(z) = \underline{\pi} + f(z)$$

$$\pi_2(z) = \overline{\pi} - z. \tag{1}$$

The function f has the properties $f(0) = 0$, $f'(0) = 1$, $0 \leq f(z) \leq z$, $f'(z) \geq 0$, $f''(z) < 0$. These assumptions ensure that smoothing is costly, in the sense that expected surplus falls as smoothing rises. Obviously we need not consider values of z above $\overline{z}$ given by $\underline{\pi}+f(\overline{z}) = \overline{\pi} - \overline{z}$.

It will turn out to be more convenient to work with an alternative representation of smoothing. Define $\lambda(z) \equiv (\pi + f(z))/(\overline{\pi} - z)$, the ratio of low to high surpluses, and define $\underline{\lambda} \equiv \lambda(0)$. Clearly λ is an increasing monotonic function of z, so we can invert it. It will be useful for later purposes to denote the marginal increase in surplus in the bad state in terms of λ rather than z, so we define the function

$$\phi(\lambda) \equiv f'(z(\lambda)). \tag{2}$$

From our assumptions on f, ϕ is a strictly decreasing function of λ.

As noted in the previous section, our assumptions about the union's operations imply that the union is concerned only with its share of the surplus in each state. We assume that both the firm and union have constant relative risk-aversion utility functions, which we write respectively as

$$v(y) = \frac{1}{1-\alpha}\, y^{1-\alpha}, \;\; u(w) = \frac{1}{1-\beta}\, w^{1-\beta} \tag{3}$$

where $0 \leq \alpha < \beta < 1$ (it avoids unnecessary complications from reservation utility to bound α and β below 1).

There are three things to be determined—the degree of smoothing (z, or equivalently, λ), and the shares of $\pi_1(z)$, $\pi_2(z)$ going to the firm. We shall think of these being bargained sequentially, with the firm and union bargaining first over z, and then over the shares. In each stage we shall represent the outcome of the bargaining process as the generalized Nash bargaining solution, and in general we may wish to allow the bargaining strength of the firm to vary between the two stages; t will denote the bargaining strength of the firm in the first stage, s in in the second stage.

We turn now to the second stage bargaining problem, and begin with *ex post bargaining*. Thus suppose that some surplus π has accrued, and the firm and union are bargaining over its division. Let $y^*(\pi)$ be the firm's pay-off, where $y^* = \operatorname{argmax}[v(y)]^s[u(\pi - y)]^{1-s}$, and let $\sigma(\pi) = y^*(\pi)/\pi$. With the utility functions in (3) it is readily checked that $\sigma(\pi) = \sigma \equiv s(1-\alpha)/[(1-s)(1-\beta) + s(1-\alpha)]$, so that the firm gets a share σ of the surplus that is independent of the size of the surplus.

With *ex ante bargaining*, we suppose that a particular level of smoothing has been selected resulting in surpluses $\pi_1(z)$, $\pi_2(z)$ with

$\lambda = \pi_1(z)/\pi_2(z)$. Then we wish to find y_1, y_2 to

$$\left.\begin{array}{ll} & \text{maximize } [V(y_1,y_2)]^s\,[U(\pi_1 - y_1,\ \pi_2 - y_2)]^{1-s} \\ \text{where} & V(y_1,y_2) = \rho v(y_1) + (1-\rho)v(y_2) \\ \text{and} & U(\pi_1 - y_1,\ \pi_2 - y_2) = \rho u(\pi_1 - y_1) + (1-\rho)u(\pi_2 - y_2). \end{array}\right\} \quad (4)$$

It is readily checked (see appendix) that (4) has a unique solution in which the firm's pay-offs can be written as

$$y_1^* = \sigma_1(\lambda)\cdot\pi_1, \quad y_2^* = \sigma_2(\lambda)\cdot\pi_2. \quad (5)$$

$\sigma_1(\lambda)$ and $\sigma_2(\lambda)$ satisfy two conditions. The first is

$$\left[\frac{\sigma_1(\lambda)\lambda}{\sigma_2(\lambda)}\right]^{\alpha} = \left[\frac{(1-\sigma_1(\lambda))\lambda}{(1-\sigma_2(\lambda))}\right]^{\beta}. \quad (6)$$

This is the condition for optimal risk sharing between the firm and the union. Define

$$\xi(\lambda) \equiv \left[\frac{\sigma_1(\lambda)\lambda}{\sigma_2(\lambda)}\right]^{\alpha} \quad (7)$$

and

$$\mu(\lambda) \equiv \frac{\rho\lambda}{\rho\lambda + (1-\rho)\xi(\lambda)}.$$

Then $\sigma_1(\lambda)$ and $\sigma_2(\lambda)$ also satisfy the second condition:

$$\sigma = \mu(\lambda)\sigma_1(y) + [1-\mu(\lambda)]\sigma_2(y). \quad (8)$$

From (6), since $\alpha < \beta$, $\lambda < 1$, $\sigma_1(\lambda) < \sigma_2(\lambda)$, and together with (8) we get $\sigma_1(y) < \sigma < \sigma_2(y)$. Thus, the firm insures the workers by taking a lower share of the surplus in the bad state and a higher share of surplus in the good state than with *ex post* bargaining.

We shall need to know how σ_1, σ_2 and ξ vary with λ. We show in the appendix that while $(d\sigma_1)/(d\lambda) > 0$, $(d\xi)/(d\lambda) > 0$, $(d\sigma_2)/(d\lambda)$ is ambiguous in sign with $(d\sigma_2)/(d\lambda) > 0$ as $\lambda \to 0$, and $(d\sigma_2)/(d\lambda) < 0$ as $\lambda \to 1$ $(d\sigma_2)/(d\lambda)$ is monotonically decreasing as λ rises. Note that for small values of λ an increase in λ will cause the firm's share in *both* states to rise.

This completes the description of the model and the derivation of the features of the wage setting stage that will be needed in later sections. We now consider solutions to the full two-stage bargaining problem.

3. Efficient contracts

In this section we characterize the properties that need to be satisfied by efficient contracts. Define

$$\mathcal{V}(z,\sigma_1,\sigma_2) \equiv \frac{\rho}{1-\alpha}\{\sigma_1[\underline{\pi}+ f(z)]\}^{1-\alpha} + \frac{1-\rho}{1-\alpha}\{\sigma_2[\overline{\pi}-z]\}^{1-\alpha}$$

$$\mathcal{U}(z,\sigma_1,\sigma_2) \equiv \frac{\rho}{1-\beta}\{(1-\sigma_1)[\underline{\pi} + f(z)]\}^{1-\beta} + \frac{1-\rho}{1-\beta}\{(1-\sigma_2)[\overline{\pi} - z]\}^{1-\beta}$$

as the expected utility for the firm and union respectively from a contract specified by (z,σ_1,σ_2). For efficiency we wish to choose (z,σ_1,σ_2) to maximize $\mathcal{V}(z,\sigma_1,\sigma_2)$ subject to $\mathcal{U}(z,\sigma_1,\sigma_2) \geq \overline{\mathcal{U}}$. It is readily shown (see Appendix B) that the necessary conditions for an efficient contract are

$$\left[\frac{\sigma_1\lambda}{\sigma_2}\right]^{\alpha} = \left[\frac{(1-\sigma_1)\lambda}{1-\sigma_2}\right]^{\beta} \tag{9}$$

and

$$\phi(\lambda) = \frac{1-\rho}{\rho}\xi(\lambda). \tag{10}$$

(9) is just the condition for efficient risk sharing (*cf.* (6)) while (10) is the condition for efficient choice of smoothing. For values of $\overline{\mathcal{U}}$ for which a contract can be found that yields $\mathcal{U}(z,\sigma_1,\sigma_2) \geq \overline{\mathcal{U}}$, the efficient contract will be unique; for ϕ is a decreasing function of λ, ξ an increasing function, so there exists a unique solution to (10) for λ, and for any λ there exists a unique *ex ante* wage contract satisfying (9) and the condition on reservation utility for the union.

An efficient contract requires that z (equivalently λ) σ_1, σ_2 be bargained over in a single contract, or equivalently that in our two-stage procedure in the second stage there be *ex ante* wage bargaining, and in the first stage the firm's bargaining strength be the same as in the second stage. The next section examines what happens when either of these conditions fails.

4. Distortions in bargaining

4.1 *Ex post* bargaining

Suppose that the firm and union were unable to sign *ex ante* wage contracts, perhaps because problems of enforceability led them to prefer the simpler *ex post* form of wage bargaining. Then obviously the condition for efficient risk sharing (9) will be violated. What effect will this have on the level of smoothing chosen? It might be thought that the absence of risk sharing through wage bargaining would induce more risk reduction through real smoothing. This turns out not to be the case in all circumstances.

The choice of real smoothing is given by maximizing

$$N(z) \equiv \left\{ \frac{\rho}{1-\alpha}[\sigma(\underline{\pi} + f(z))]^{1-\alpha} + \frac{1-\rho}{1-\alpha}[\sigma(\overline{\pi} - z)]^{1-\alpha} \right\}^{s} \times \left\{ \frac{\rho}{1-\beta}[(1-\sigma)(\underline{\pi} + f(z))]^{1-\beta} + \frac{1-\rho}{1-\beta}[(1-\sigma)(\overline{\pi} - z)]^{1-\beta} \right\}^{1-s}. \tag{11}$$

Note first that the optimal choice of z is independent of σ. The first-order condition for the maximization of (11) can be written as

$$s(1-\alpha)\left[\frac{\rho\pi_1^{-\alpha} f'(z) - (1-\rho)\pi_2^{-\alpha}}{\rho\pi_1^{1-\alpha} + (1-\rho)\pi_2^{1-\alpha}}\right] + (1-s)(1-\beta)\left[\frac{\rho\pi_1^{-\beta} f'(z) - (1-\rho)\pi_2^{-\beta}}{\rho\pi_1^{1-\beta} + (1-\rho)\pi_2^{1-\beta}}\right] = 0$$

i.e.

$$s(1-\alpha)\left[\frac{\rho f'(z) - (1-\rho)\lambda^{\alpha}}{\rho + (1-\rho)\lambda^{\alpha-1}}\right] + (1-s)(1-\beta)\left[\frac{\rho f'(z) - (1-\rho)\lambda^{\beta}}{\rho + (1-\rho)\lambda^{\beta-1}}\right] = 0. \tag{12}$$

Define

$$\tau_1(\lambda) = \frac{s(1-\alpha)}{\rho + (1-\rho)\lambda^{\alpha-1}},\ \tau_2(\lambda) = \frac{(1-s)(1-\beta)}{\rho + (1-\rho)\lambda^{\beta-1}},$$

$$\tau(\lambda) = \frac{\tau_1(\lambda)}{\tau_1(\lambda) + \tau_2(\lambda)}. \tag{13}$$

Then (12) becomes

$$f'(z) = \frac{1-\rho}{\rho}[\tau(\lambda)\lambda^{\alpha} + (1-\tau(\lambda))\lambda^{\beta}]$$

or

$$\phi(\lambda) = \frac{1-\rho}{\rho}\psi(\lambda) \tag{14}$$

where

$$\psi(\lambda) \equiv \tau(\lambda)\lambda^{\alpha} + [1 - \tau(\lambda)]\lambda^{\beta}.$$

It is readily checked that $\tau(\lambda)$ rises monotonically from 0 to σ as λ rises from 0 to 1, and hence that $\psi(\lambda)$ is a monotonically increasing function of λ.

Defining $B_1(\lambda) \equiv \psi(\lambda)/\xi(\lambda)$, a comparison of (14) and (10) shows that with *ex post* bargaining the level of real smoothing will be below the efficient level iff $B_1(\lambda) > 1$, and above the efficient level if $B_1(\lambda) < 1$. Recalling that

$$\xi(\lambda) = \left[\frac{\sigma_1(\lambda)\lambda}{\sigma_2(\lambda)}\right]^{\alpha} \lambda^{\alpha} = \left[\frac{(1 - \sigma_1(\lambda))\lambda}{(1 - \sigma_2(\lambda))}\right]^{\beta} \lambda^{\beta},$$

we can write

$$B_1(\lambda) = \tau(\lambda)\left[\frac{\sigma_2(\lambda)}{\sigma_1(\lambda)}\right]^{\alpha} + (1 - \tau(\lambda))\left[\frac{(1 - \sigma_2(\lambda))}{(1 - \sigma_1(\lambda))}\right]^{\beta} \qquad (15)$$

so $B_1(\lambda)$ is a weighted average of a term that is greater than 1 and one that is less than 1, where the weight on the former term never exceeds σ. While the weight on the term $[\sigma_2/\sigma_1]^{\alpha}$ increases as λ increases, σ_2/σ_1 itself declines monotonically. It has not proved possible to determine analytically for which values of λ, $B_1(\lambda)$ will be greater or less than 1, and we present some numerical results on this in the next section.

4.2 Differences in bargaining strength

The second distortion we consider is where there is *ex ante* wage bargaining, but the firm's bargaining strength in the bargain over $z(\lambda)$ differs from that in the bargain over wages, being t in the first case, s in the second. To the extent that this is supposed to reflect an inability of unions to commit themselves to long-term contracts (Grout 1984), then we would suppose that the firm would have more say in determining z than in determining wages, so that t would be greater than s. However, we leave the direction of the distortion open.

For any given choice of z, $\sigma_1(z)$, $\sigma_2(z)$ satisfying equations (6) and (8) solve the *ex ante* wage bargaining problem, and it is shown in Appendix C that the condition for determining the bargained choice of $z(\lambda)$ is:

$$\phi(\lambda) = \frac{(1 - \rho)\xi\left[\frac{t}{s}\,\sigma_2 + \frac{1 - t}{1 - s}(1 - \sigma_2)\right] - \lambda\eta\left[\frac{t}{s} - \frac{1 - t}{1 - s}\right]}{\rho\left[\frac{t}{s}\,\sigma_1 + \frac{1 - t}{1 - s}(1 - \sigma_1)\right] + \eta\left[\frac{t}{s} - \frac{1 - t}{1 - s}\right]} \qquad (16)$$

where $\eta(\lambda) = \rho\lambda(d\sigma_1)/(d\lambda) + (1-\rho)\xi(d\sigma_2)/(d\lambda)$.

Define

$$\chi(\lambda) = \frac{\xi\left[\frac{t}{s}\,\sigma_2 + \frac{1-t}{1-s}(1-\sigma_2)\right] - \frac{\lambda\eta}{1-\rho}\left[\frac{t}{s} - \frac{1-t}{1-s}\right]}{\left[\frac{t}{s}\,\sigma_1 + \frac{1-t}{1-s}(1-\sigma_1)\right] + \frac{\eta}{\rho}\left[\frac{t}{s} - \frac{1-t}{1-s}\right]}$$

then we can write (16) as

$$\phi(\lambda) = \frac{1-\rho}{\rho}\chi(\lambda). \tag{17}$$

Define $B_2(\lambda) = \chi(\lambda)/\xi(\lambda)$, then comparison with (10) shows that we will get less than the efficient level of smoothing if $B_2(\lambda) > 1$, and more if $B_2(\lambda) < 1$.

To investigate the possible directions of the bias, it is simplest to begin with the case where $\eta = 0$. Then

$$B_2(\lambda) = \frac{\frac{t}{s}\,\sigma_2 + \frac{1-t}{1-s}(1-\sigma_2)}{\frac{t}{s}\,\sigma_1 + \frac{1-t}{1-s}(1-\sigma_1)}$$

and it is readily shown that $B_2(\lambda) \gtreqless 1$ according as $t \gtreqless s$. To rationalize this result, note that a sufficient condition for $\eta = 0$ is that $(d\sigma_1)/(d\lambda) = (d\sigma_2)/(d\lambda) = 0$. So suppose that *ex ante* bargaining just leads to setting some σ_1, σ_2 with $0 \le \sigma_1 \le \sigma \le \sigma_2 \le 1$ but with σ_1, σ_2 not varying with λ, so there is no gain to the firm or union from varying λ in terms of its effect on their share of the subsequent wage bargaining process. Then, if the firm has more say in the level of smoothing, it is going to want to reduce the amount of smoothing ($B_2(\lambda) > 1$), because it gets a low share of the return to smoothing and pays a high share of the cost of smoothing, while the situation is reversed for the union, and so it will want to do more than the efficient level of smoothing if it has a greater say in the choice of smoothing.

Next note that if $\eta < 0$, this would reinforce the above argument, making $B_2(\lambda)$ bigger if $t > s$, or smaller if $t < s$. So for $\eta \le 0$ the direction of bias is unambiguous. Thus it is only for the case $\eta > 0$ that there will be ambiguity. As we show in Appendix A, this is precisely the case that arises. However, it is also clear from Appendix A that as $\lambda \to 1$, $\eta \to 0$ (recall that $(d\sigma_1)/(d\lambda) > 0$ but for large λ $(d\sigma_2)/(d\lambda) < 0$) so that for large values of λ, the bias will be as predicted for the case $\eta = 0$. When λ is small, then we have both

$(d\sigma_1)/(d\lambda) > 1$ and $(d\sigma_2)/(d\lambda) > 0$, and so if the effect of η is going to offset the prediction for the case $\eta = 0$, it is most likely to do so for small values of λ.

4.3 Combined sources of distortion

It is straightforward to combine both sources of distortion. If there is *ex post* bargaining over wages and differences in bargaining strength between the two stages, then in (13), the equations defining $\tau_1(\lambda)$ and $\tau_2(\lambda)$, s is replaced by t, and with this change (14) remains the condition for the bargained choice of λ. The expression for the resulting bias will be the same as for $B_1(\lambda)$ in (15), except that again the expression for $\tau(\lambda)$ will depend on t not s. In general, the direction of bias is ambiguous, but for extreme values of t it can be easily determined. For $t = 1$, $\tau = 1$, and so

$$B_1(\lambda) = \left[\frac{\sigma_2(\lambda)}{\sigma_1(\lambda)}\right]^{\alpha} \geq 1$$

for all λ, while for $t = 0$, $B_1(\lambda) \leq 1$ for all λ. Thus if the firm chooses smoothing and there is *ex post* bargaining, there will always be less than the efficient level of smoothing.

5. Some numerical results

Except for the extreme cases discussed at the end of §4, it has not proved possible analytically to derive the direction of bias from the efficient level of smoothing that results from different distortions to the wage bargaining process, although, at least for the second source of bias, we could show that the ambiguity would arise only from low values of λ. To get some feel for the direction of biases involved in the two cases, we carried out some numerical calculations. We took values for the parameters α, β, ρ, s, t and calculated $B_1(\lambda)$, $B_2(\lambda)$ for values of λ ranging from 0.01 to 0.95. Each parameter took the values 0.1, 0.3, 0.5, 0.7, 0.9 and we analysed all possible combinations of parameters, with the proviso that $\beta > \alpha$.

The patterns that emerged for $B_1(\lambda)$ and $B_2(\lambda)$ (for the case $t > s$) are shown in Figs. 1(a) and (b). (The pattern for $B_2(\lambda)$ for $t < s$ would be the the reflection of that shown around $B_2(\lambda) = 1.0$.) The broad conclusion is that for most values of λ, $B_1(\lambda) < 1.0$ and $B_2(\lambda) > 1.0$ (for $t > s$). The converse cases only occur for small values of λ, and if we let $\bar{\lambda}_1$, $\bar{\lambda}_2$ be the values of λ for which $B_1(\bar{\lambda}_1) = 1.0$, $B_2(\bar{\lambda}_2) = 1.0$ respectively, then for a broad range of parameter values these would be small (less than 0.05) and in many cases 0.0. Only

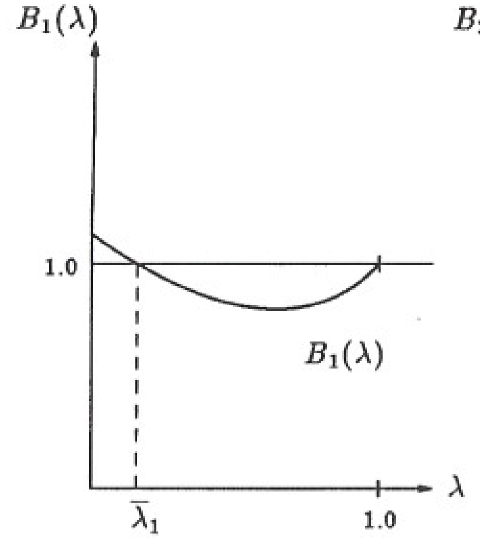

Figure 1(a)
First Source of Bias

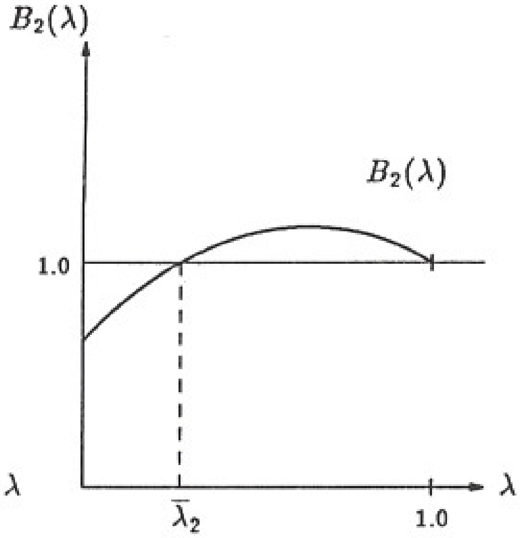

Figure 1(b)
Second Source of Bias $(t > s)$

for values of α and β close together and small (e.g. $\alpha = 0.1$, and $\beta = 0.3$), and small values of ρ (e.g. $\rho < 0.5$) would $\overline{\lambda}_1$ and $\overline{\lambda}_2$ become significant (e.g. for $\alpha = 0.1$, $\beta = 0.3$, $\rho = 0.1$, $s = 0.3$, $t = 0.7$). $\overline{\lambda}_1$ never took a value above 0.16, $\overline{\lambda}_2$ never took a value above 0.31. Thus if we assume, quite plausibly, that $\underline{\lambda}$, the ratio of surplus in the bad state to surplus in the good state with no smoothing, took a value above 0.31, then we could say unambiguously that *ex post* rather than *ex ante* wage bargaining would lead to too high a level of smoothing, while giving more bargaining strength to the firm (union) in choice of smoothing will lead to too little (much) smoothing, all comparison being with the efficient level of smoothing.

We have also attempted to assess the welfare consequences of the distortions from efficient bargaining. To do this requires us to specify a particular function for f (equivalently ϕ) and compute the efficient outcome and an inefficient outcome, and then compare the resulting levels of utility for the firm and union under the two outcomes. For ease of computation we took as our example of an inefficient contract the case of *ex post* bargaining over wages when the firm chooses the level of smoothing ($t = 1$). For the function f we chose one which was piecewise linear, shown in Fig. 2(a), leading to a step function for ϕ shown in Fig. 2(b).

More precisely, we proceeded as follows. The function $f(z)$ can be written as

$$f(z) = C_2 \cdot z \qquad z \leq z_0$$

$$f(z) = d_1 + C_1 \qquad z \geq z_0.$$

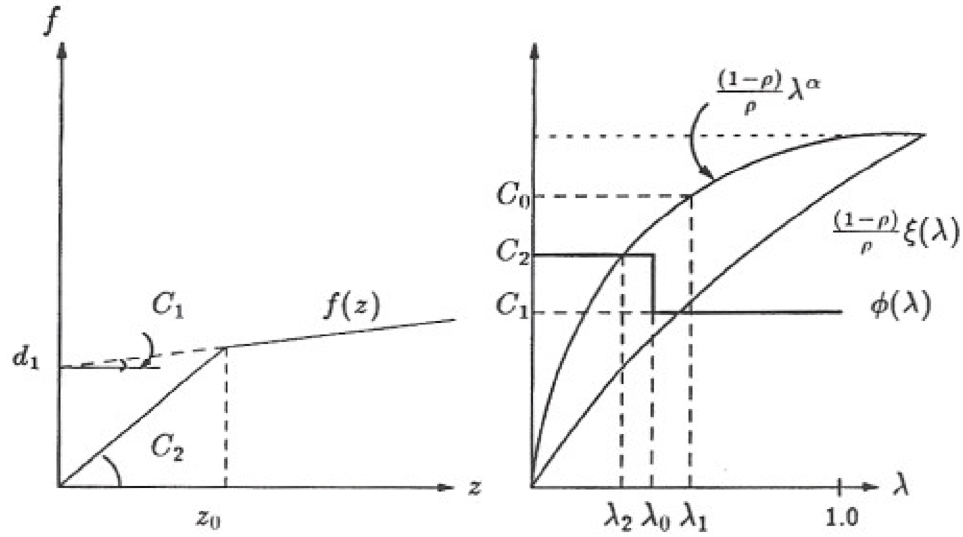

Figure 2(a) **Figure 2(b)**

We need to set the parameters C_1, C_2, d_1, z_0. To do this, for some given value of λ, say λ_1, we first set $C_1 = (1-\rho/\rho)\xi(\lambda_1)$, ensuring that λ_1 would be the efficient level of smoothing. We then calculated $C_0 = (1-\rho/\rho)\lambda_1^\alpha$, an upper bound on C_2, and set $C_2 = (C_0 + C_1)/2$. Knowing C_2 we could then readily calculate λ_2, the inefficient level of smoothing, from $(1-\rho/\rho)\lambda_2^\alpha = C_2$. To choose the switch point we set $\lambda_0 = (\lambda_1 + \lambda_2)/2$, and then translated λ_0, λ_1, λ_2 into z_0, z_1, z_2 respectively, and chose d_1 to ensure that $C_2 z_0 = d_1 + C_1 z_0$. From the rest of the parameters, we could readily calculate the wage bargain that would be struck in the efficient and inefficient contracts, and from that calculate the levels of expected utility for the firm and union derived from the contracts. To express these measures of welfare in more interpretable units, we converted expected utility into the corresponding certainty equivalent income, for both the efficient and inefficient contracts. Since the biases from distortions in the bargaining process become small as λ tends to 1, the welfare effects turn out to be most significant for small values of λ, and are also greater when α and β are quite different, for then there are substantial gains to efficient risk sharing. As an illustration of the orders of magnitude involved, we calculated the ratios of certainty equivalent income with the inefficient contract to that with the efficient contract for the firm, union, and in total for values of λ between 0.05 and 0.3, for parameter values $\alpha = 0.1$, $\beta = 0.9$, $\rho = 0.5$, $s = 0.25$, $t = 1.0$, and these are presented in Table 1.

Table 1. Welfare losses for inefficient contracts

	Ratio of Certain Equivalent Income from Inefficient Contract to that for Efficient Contract		
λ	Union	Firm	Total
0.05	0.25	0.98	0.85
0.10	0.51	0.99	0.89
0.15	0.66	1.00	0.92
0.20	0.75	1.00	0.94
0.25	0.80	1.00	0.95
0.30	0.85	1.00	0.96

As λ increases beyond 0.3 the welfare losses continue to decline. One feature which emerges immediately is that, for the particular form of inefficiency used here, the costs of inefficiency can be quite significant in total, but are borne overwhelmingly by the union. This is not surprising, since we have given the firm the power to choose the degree of smoothing that it likes best, so all that it is faced with is the loss from not having efficient risk sharing and, since the firm is almost risk neutral, these losses are small.

To summarize, then, the numerical calculations show that for large values of λ ($\lambda > 0.3$) the biases caused by distortion in the bargaining process can be clearly identified—the use of *ex post* bargaining will lead to excessive smoothing, while giving different bargaining strengths between the two stages of bargaining will lead to too little or too much smoothing, depending on when the firm or the union has more bargaining power in the bargain over smoothing than it has in the bargain over wages. Both these conclusions can be reversed for small values of λ. We have also shown that for a particular combination of inefficiencies, the welfare costs from inefficiency can be quite large, though the welfare cost is concentrated almost entirely on the union.

6. Conclusions

In this paper we have argued that differences in the wage setting process can affect the way in which an economy may respond to shocks because differences in the way in which the wage bargaining process allows workers to obtain protection against shocks will affect the extent to which shocks are smoothed in real terms. We have demonstrated this in the context of an extremely simple model, for which the conventional implicit contract story would provide no explanation for differences in the response of an economy to shocks arising from differences in wage setting. We have considered the impact of varying the extent to which workers and firms can make contingent wage (employment) contracts, and the extent to which workers are involved in decisions on how much real smoothing takes place. Differences in these institutional features from one economy to another will cause the extent to which the economy tries to smooth out such shocks to vary, and, at least for plausible values of the magnitude of such shocks, we can identify which direction these differences will bias the level of smoothing for its efficient level.

Obviously the model we have used is extremely simple, but even for such a simple model it has not proved possible to derive analytically all the comparisons across different wage bargaining structures. Nevertheless, the model does establish the point we sought to make. We would expect that in a richer model the effects we have analysed here would become more pronounced. In particular we would hope to extend the model to allow for situations where unions care about employment as well as income (so unions do not insure their members against unemployment), and for asymmetric information in second stage bargains.

Appendix A. Properties of $\sigma_1(\lambda)$, $\sigma_2(\lambda)$, $\xi(\lambda)$ and $\eta(\lambda)$

1. Determination of $\sigma_1(\lambda)$ and $\sigma_2(\lambda)$

Choose y_1, y_2 to

$$\max_{y_1, y_2} N(y_1, y_2) \equiv [V(y_1, y_2)]^s \{U[(\pi_1 - y_1), (\pi_2 - y_2)]\}^{1-s}$$

where

$$V(y_1, y_2) = \rho \frac{y_1^{1-\alpha}}{1-\alpha} + (1-\rho)\frac{y_2^{1-\alpha}}{1-\alpha}$$

$$U(\pi_1 - y_1, \pi_2 - y_2) = \frac{\rho(\pi_1 - y_1)^{1-\beta}}{1-\beta} + (1-\rho)\frac{(\pi_2 - y_2)^{1-\beta}}{1-\beta}.$$

First-order conditions are

$$\frac{\partial N}{\partial y_1} = \frac{s\rho y_1^{-\alpha}}{V} - \frac{(1-s)\rho(\pi_1 - y_1)^{-\beta}}{U} = 0 \tag{A1}$$

$$\frac{\partial N}{\partial y_2} = \frac{s(1-\rho)y_1^{-\alpha}}{V} - \frac{(1-s)(1-\rho)(\pi_2 - y_2)^{-\beta}}{U} = 0 \qquad \text{(A2)}$$

$$\text{(A1)/(A2)} \Rightarrow \left[\frac{y_1}{y_2}\right]^{\alpha} = \left[\frac{\pi_1 - y_1}{\pi_2 - y_2}\right]^{\beta}. \qquad \text{(A3)}$$

Defining $\sigma_1 = y_1/\pi_1$, $\sigma_2 = y_2/\pi_2$, (A3) becomes

$$\left[\frac{\sigma_1 \lambda}{\sigma_2}\right]^{\alpha} = \left[\frac{(1-\sigma_1)\lambda}{(1-\sigma_2)}\right]^{\beta} \qquad \text{(A4)}$$

which is equation (6) in the text. Define $\xi \equiv [\sigma_1 \lambda/\sigma_2]^{\alpha} = [(1-\sigma_1)\lambda/(1-\sigma_2)]^{\beta}$. (A1) can be rewritten as

$$\frac{s(1-\alpha)}{(1-s)(1-\beta)} = \frac{\rho y_1 + (1-\rho) y_2 \left[\frac{y_1}{y_2}\right]^{\alpha}}{\rho(\pi_1 - y_1) + (1-\rho)(\pi_2 - y_2)\left[\frac{\pi_1 - y_1}{\pi_2 - y_2}\right]^{\beta}} \qquad \text{(A5)}$$

$$\frac{s(1-\alpha)}{(1-s)(1-\beta)} = \frac{\rho\lambda\sigma_1 + (1-\rho)\xi\sigma_2}{\rho\lambda(1-\rho_1) + (1-\rho)\xi(1-\sigma_2)}. \qquad \text{(A6)}$$

A little manipulation yields

$$\frac{s(1-\alpha)}{s(1-\alpha) + (1-s)(1-\beta)} = \left[\frac{\rho\lambda}{\rho\lambda + (1-\rho)\xi}\right]\sigma_1 + \left[\frac{(1-\rho)\xi}{\rho\lambda + (1-\rho)\xi}\right]\sigma_2$$

$$\sigma = \mu \cdot \sigma_1 + (1-\mu)\sigma_2 \qquad \text{(A7)}$$

where $\mu = \dfrac{\rho\lambda}{\rho\lambda + (1-\rho)\xi}$.

(A7) is equation (8) in the text.

2. Determination of $d\sigma_1/d\lambda$, $d\sigma_2/d\lambda$

Rewrite (A4) as

$$\frac{\sigma_1^{\alpha}}{(1-\sigma_1)^{\beta}} = \lambda^{\beta-\alpha} \frac{\sigma_2^{\alpha}}{(1-\sigma_2)^{\beta}}. \qquad \text{(A8)}$$

Totally differentiate (A8) with respect to λ to obtain

$$\left[\frac{\alpha}{\sigma_1} + \frac{\beta}{1-\sigma_1}\right]\frac{d\sigma_1}{d\lambda} = \frac{\beta-\alpha}{\lambda} + \left[\frac{\alpha}{\sigma_2} + \frac{\beta}{1-\sigma_2}\right]\frac{d\sigma_2}{d\lambda}$$

$$\text{or} \qquad A_{11}\frac{d\sigma_1}{d\lambda} + A_{12}\frac{d\sigma_2}{d\lambda} = B_1 \qquad \text{(A9)}$$

where

$$A_{11} = \frac{\alpha}{\sigma_1} + \frac{\beta}{1-\sigma_1} > 0, \quad A_{12} = -\left[\frac{\alpha}{\sigma_2} + \frac{\beta}{1-\sigma_2}\right] < 0$$

$$B_1 = \frac{\beta - \alpha}{\lambda} > 0.$$

(A7) can be rewritten as

$$\sigma[\rho\lambda + (1-\rho)\xi] = \rho\lambda\sigma_1 + (1-\rho)\xi\sigma_2. \tag{A10}$$

Differentiating (A10) totally with respect to λ we have

$$(\sigma - \sigma_1)\rho = (1-\rho)(\sigma_2 - \sigma)\frac{d\xi}{d\lambda} + \rho\lambda\frac{d\sigma_1}{d\lambda} + (1-\rho)\xi\frac{d\sigma_2}{d\lambda} \tag{A11}$$

$$\frac{d\xi}{d\lambda} = \alpha\xi\left[\frac{1}{\lambda} - \frac{1}{\sigma_1}\,\frac{d\sigma_1}{d\lambda} - \frac{1}{\sigma_2}\cdot\frac{d\sigma_2}{d\lambda}\right]. \tag{A12}$$

Putting (A12) into (A11), using (A10) and collecting terms we have

$$A_{21}\,\frac{d\sigma_1}{d\lambda} + A_{22}\frac{d\sigma_2}{d\lambda} = B_2 \tag{A13}$$

where

$$A_{21} = \frac{\rho\lambda}{\sigma_1}\,[(1-\alpha)\sigma_1 + \alpha\sigma] > 0$$

$$A_{22} = \frac{(1-\rho)\xi}{\sigma_2}\,[(1-\alpha)\sigma_2 + \alpha\sigma] > 0$$

$$B_2 = \rho(1-\alpha)(\sigma - \sigma_1) > 0.$$

From (A9) and (A13)

$$\frac{d\sigma_1}{d\lambda} = \frac{A_{22}B_1 - A_{12}B_2}{\Delta}$$

$$\frac{d\sigma_2}{d\lambda} = \frac{A_{11}B_2 - A_{21}B_1}{\Delta}$$

where $\Delta = A_{11}A_{22} - A_{12}A_{21} > 0$.

Clearly, $\frac{d\sigma_1}{d\lambda} > 0$. By definition of A_{11}, B_2, A_{21}, B_1,

$$\Delta \frac{d\sigma_2}{d\lambda} = \left[\frac{\alpha}{\sigma_1} + \frac{\beta}{1-\sigma_1}\right]\rho(1-\alpha)(\sigma-\sigma_1)$$

$$- \frac{\rho}{\sigma_1}[(1-\alpha)\sigma_1 + \alpha\sigma](\beta-\alpha)$$

$$\frac{\Delta}{\rho}\frac{d\sigma_2}{d\lambda} = \alpha(1-\alpha)\frac{(\sigma-\sigma_1)}{\sigma_1} + \beta\frac{(1-\alpha)(\sigma-\sigma_1)}{1-\sigma_1}$$

$$-(\beta-\alpha)(1-\alpha) - \alpha(\beta-\alpha)\frac{\sigma}{\sigma_1}.$$

A little manipulation then yields

$$\text{sign}\,\frac{d\sigma_2}{d\lambda} = \text{sign}\left\{\alpha(1-\beta)\frac{\sigma}{\sigma_1} - \beta(1-\alpha)\frac{(1-\sigma)}{1-\sigma_1}\right\}. \qquad \text{(A14)}$$

Now as $\lambda \to 1$, $\sigma_1 \to \sigma$, and RHS of (A14) $\to \alpha - \beta < 0$. On the other hand, if as $\lambda \to 0$, $\sigma_1 \to 0$, then RHS of (A14) $\to +\infty$. So for larger values of λ, $d\sigma_2/d\lambda$ will be $-ve$, but for small values of λ, $d\sigma_2/d\lambda$ could be $+ve$.

3. Sign of $d\xi/d\lambda$

From (A12) a sufficient condition of $d\xi/d\lambda > 0$ is that $X \equiv \sigma_2(d\sigma_1/d\lambda) - \sigma_1(d\sigma_2/d\lambda) > 0$. From the expressions for $d\sigma_1/d\lambda$, $d\sigma_2/d\lambda$,

$$\Delta X = \sigma_2 A_{22} B_1 - \sigma_2 A_{12} B_2 - \sigma_1 A_{11} B_2 + \sigma_1 A_{21} B_1$$

$$= B_1(\sigma_2 A_{22} + \sigma_1 A_{21}) + B_2\left\{\sigma_2\left[\frac{\alpha}{\sigma_2} + \frac{\beta}{1-\sigma_2}\right] - \sigma_1\left[\frac{\alpha}{\sigma_1} + \frac{\beta}{1-\sigma_1}\right]\right\}$$

$$= B_1(\sigma_2 A_{22} + \sigma_1 A_{21}) + \beta B_2\left[\frac{\sigma_2}{1-\sigma_2} - \frac{\sigma_1}{1-\sigma_1}\right] > 0.$$

4. Sign of η

$$\eta = \rho\lambda\frac{d\sigma_1}{d\lambda} + (1-\rho)\xi\frac{d\sigma_2}{d\lambda}$$

$$\Delta\eta = \rho\lambda[A_{22}B_1 - A_{12}B_2] + (1-\rho)\xi[A_{11}B_2 - A_{21}B_1]$$

$$= B_2[(1-\rho)\xi A_{11} - \rho\lambda A_{12}] + B_1[\rho\lambda A_{22} - (1-\rho)\xi A_{21}]$$

$$= \rho(1-\alpha)(\sigma-\sigma_1)\left[(1-\rho)\xi\left(\frac{\alpha}{\sigma_1} + \frac{\beta}{1-\sigma_1}\right) + \rho\lambda\left(\frac{\alpha}{\sigma_2} + \frac{\beta}{1-\sigma_2}\right)\right]$$

$$+ \frac{(\beta-\alpha)}{\lambda}\left\{\rho\lambda(1-\rho)\xi\left[1-\alpha+\alpha\frac{\sigma}{\sigma_2}\right] - (1-\rho)\xi\rho\lambda\left[1-\alpha+\alpha\frac{\sigma}{\sigma_1}\right]\right\}.$$

Using (A10)

$$\Delta\eta = \rho(1-\rho)\xi(1-\alpha)\left[(\sigma-\sigma_1)\left(\frac{\alpha}{\sigma_1}+\frac{\beta}{1-\sigma_1}\right)+(\sigma_2-\sigma)\left(\frac{\alpha}{\sigma_2}+\frac{\beta}{1-\sigma_2}\right)\right]$$
$$+(\beta-\alpha)\rho(1-\rho)\xi\alpha\left[\frac{\sigma}{\sigma_2}-\frac{\sigma}{\sigma_1}\right]$$
$$=\rho(1-\rho)\xi\left\{(1-\alpha)\alpha\left(\frac{\sigma}{\sigma_1}-1\right)+\beta(1-\alpha)\left(\frac{\sigma-\sigma_1}{1-\sigma_1}\right)+(1-\alpha)\alpha\left[1-\frac{\sigma}{\sigma_2}\right]\right.$$
$$\left.+(1-\alpha)\beta\left(\frac{\sigma_2-\sigma}{1-\sigma_2}\right)+\alpha(\beta-\alpha)\frac{\sigma}{\sigma_2}-\alpha(\beta-\alpha)\frac{\sigma}{\sigma_1}\right\}$$
$$=\rho(1-\rho)\xi\left\{\alpha(1-\beta)\left(\frac{\sigma}{\sigma_1}-\frac{\sigma}{\sigma_2}\right)+\beta(1-\alpha)\left[\frac{\sigma-\sigma_1}{1-\sigma}+\frac{\sigma_2-\sigma}{1-\sigma_2}\right]\right\}\geq 0.$$

Since $\Delta > 0$, $\eta \geq 0$.

Appendix B. Derivation of efficient contracts

Define

$$\mathcal{V}(z,\sigma_1,\sigma_2)\equiv\frac{\rho}{1-\alpha}[\sigma_1(\underline{\pi}+f(z))]^{1-\alpha}+\frac{1-\rho}{1-\alpha}[\sigma_2(\overline{\pi}-z))]^{1-\alpha}$$

$$\mathcal{U}(z_1,\sigma_1,\sigma_2)\equiv\frac{\rho}{1-\beta}[(1-\sigma_1)(\underline{\pi}+f(z))]^{1-\beta}+\frac{1-\rho}{1-\beta}[(1-\sigma_2)\overline{\pi}-z)]^{1-\beta}$$

the expected utility of the firm and union respectively for contract (z_1,σ_1,σ_2). For efficiency we seek a contract to maximize $\mathcal{V}(z,\sigma_1,\sigma_2)$ subject to $\mathcal{U}(z,\sigma_1,\sigma_2)\geq\overline{\mathcal{U}}$. Set up the Lagrangian $\mathcal{L}(z,\sigma_1,\sigma_2)=\mathcal{V}(z,\sigma_1,\sigma_2)+\nu[\mathcal{U}(z,\sigma_1,\sigma_2)-\overline{\mathcal{U}}]$. Then, first-order conditions for efficiency (at an interior solution) are

$$\frac{\partial\mathcal{L}}{\partial\sigma_1}=\rho\sigma_1^{-\alpha}[\underline{\pi}+f(z)]^{1-\alpha}-\nu\rho(1-\sigma_1)^{-\beta}(\underline{\pi}+f(z))^{1-\beta}=0 \quad \text{(B1)}$$

$$\frac{\partial\mathcal{L}}{\partial\sigma_2}=(1-\rho)\sigma_2^{-\alpha}[\overline{\pi}-z]^{1-\alpha}-\nu(1-\rho)(1-\sigma_2)^{-\beta}[\overline{\pi}-z]^{1-\beta}=0 \quad \text{(B2)}$$

$$\frac{\partial\mathcal{L}}{\partial z}=\rho[\sigma_1(\underline{\pi}+f(z))]^{\alpha}\sigma_1 f'(z)-(1-\rho)[\sigma_2(\overline{\pi}-z)]^{-\alpha}\sigma_2$$
$$+\nu\rho[(1-\rho)(\underline{\pi}+f(z))]^{-\beta}(1-\sigma_1)f'(z)$$
$$-\nu(1-\rho)[(1-\sigma_2)(\overline{\pi}-z)]^{-\beta}](1-\sigma_2)=0 \quad \text{(B3)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathcal{U}(z, \sigma_1, \sigma_2) - \overline{\mathcal{U}} = 0 \tag{B4}$$

$$(\mathrm{B2/B1}) \Rightarrow \left(\frac{\sigma_1 \lambda}{\sigma_2}\right)^{\alpha} = \left[\frac{(1-\sigma_1)\lambda}{\sigma_2}\right]^{\beta}. \tag{B5}$$

From (B3)

$$f'(z) = \frac{1-\rho}{\rho}\left\{\frac{\sigma_2^{1-\alpha}\pi_2^{-\alpha} + \nu(1-\sigma_2)^{1-\beta}\pi_2^{-\beta}}{\sigma_1{}^{1-\alpha}\pi_1^{-\alpha} + \nu(1-\sigma_1)^{1-\beta}\pi_1^{-\beta}}\right\}. \tag{B6}$$

From (B1) $\nu(1-\sigma_1)^{-\beta}\pi_1^{-\beta} = \sigma_1^{-\alpha}\pi_1^{-\alpha}$, and (B2) $\nu(1-\sigma_2)^{-\beta}\pi_2^{-\beta} = \sigma_2^{-\alpha}\pi_2^{-\alpha}$ and inserting in (B6) yields

$$f'(z) = \frac{1-\rho}{\rho}\left[\frac{\sigma_1 \lambda}{\sigma_2}\right]^{\alpha} = \frac{1-\rho}{\rho}\,\xi. \tag{B7}$$

So the condition for efficient choice of smoothing is

$$\phi(\lambda) = \frac{1-\rho}{\rho}\,\xi(\lambda).$$

Appendix C. Determination of bargained level of smoothing

Define

$$\widehat{V}(\pi_1(z),\pi_2(z)) \equiv \rho v\left[\sigma_1\left(\frac{\pi_1}{\pi_2}\right)\cdot \pi_1\right] + (1-\rho)v\left[\sigma_2\left(\frac{\pi_1}{\pi_2}\right)\cdot \pi_2\right]$$

$$\widehat{U}(\pi_1(z),\pi_2(z)) \equiv \rho u\left[\left(1-\sigma_1\left(\frac{\pi_1}{\pi_2}\right)\right)\cdot \pi_1\right] + (1-\rho)u\left[\left(1-\sigma_2\left(\frac{\pi_1}{\pi_2}\right)\right)\cdot \pi_2\right]$$

$$\widehat{N}(\pi_1(z),\pi_2(z)) \equiv [\widehat{V}]^t[\widehat{U}]^{1-t}.$$

We choose z to maximize $\widehat{N}$. The first-order condition is

$$0 = \frac{t}{\widehat{V}}\cdot\frac{d\widehat{V}}{dz} + \frac{(1-t)}{\widehat{U}}\cdot\frac{d\widehat{U}}{dz} \tag{C1}$$

where

$$\left.\begin{aligned} \frac{d\widehat{V}}{dz} &= \frac{\partial \widehat{V}}{\partial \pi_1}\cdot f'(z) - \frac{\partial \widehat{V}}{\partial \pi_2} \\ \frac{d\widehat{U}}{dz} &= \frac{\partial \widehat{U}}{\partial \pi_1}\cdot f'(z) - \frac{\partial \widehat{U}}{\partial \pi_2}. \end{aligned}\right\} \tag{C2}$$

Putting (C2) into (C1) and rearranging yields

$$f'(z) = \left\{ t\frac{\frac{\partial \widehat{V}}{\partial \pi_2}}{\widehat{V}} + (1-t)\frac{\frac{\partial \widehat{U}}{\partial \pi_2}}{\widehat{U}} \right\} \Big/ \left\{ t\frac{\frac{\partial \widehat{V}}{\partial \pi_1}}{\widehat{V}} + (1-t)\frac{\frac{\partial \widehat{U}}{\partial \pi_1}}{\widehat{U}} \right\} \qquad \text{(C3)}$$

$$\frac{\partial \widehat{V}}{\partial \pi_1} = \rho v'(\sigma_1 \pi_1)\sigma_1 + \left[\rho v'(\sigma_1\pi_1)\pi_1 \frac{d\sigma_1}{d\lambda} + (1-\rho)v'(\sigma_2\pi_2)\pi_2\frac{d\sigma_2}{d\lambda}\right]\frac{\partial \lambda}{\partial \pi_1}$$

$$= \rho(\sigma_1\pi_1)^{-\alpha}\sigma_1 + \left[\rho(\sigma_1\pi_1)^{-\alpha}\pi_1\frac{d\sigma_1}{d\lambda} + (1-\rho)(\sigma_2\pi_2)^{-\alpha}\pi_2\frac{d\sigma_2}{d\lambda}\right]\cdot\frac{1}{\pi_2}$$

$$\frac{\partial \widehat{V}}{\partial \pi_1} = (\sigma_1\pi_1)^{-\alpha}[\rho\sigma_1 + \eta] \qquad \text{(C4)}$$

where η is defined by

$$\eta = \left[\rho\lambda\frac{d\sigma_1}{d\lambda} + (1-\rho)\xi\frac{d\sigma_2}{d\lambda}\right].$$

Similarly we find that

$$\frac{\partial \widehat{V}}{\partial \pi_2} = (\sigma_1\pi_1)^{-\alpha}[(1-\rho)\xi\sigma_2 - \lambda\eta] \qquad \text{(C5)}$$

$$\widehat{V} = \frac{(\sigma_1\pi_1)^{-\alpha}\pi_2}{(1-\alpha)}[\rho\lambda\sigma_1 + (1-\rho)\xi\sigma_2] \qquad \text{(C6)}$$

$$\frac{\partial \widehat{U}}{\partial \pi_1} = [(1-\sigma_1)\pi_1]^{-\beta}[\rho(1-\sigma_1) - \eta] \qquad \text{(C7)}$$

$$\frac{\partial \widehat{U}}{\partial \pi_2} = [(1-\sigma_1)\pi_1)^{-\beta}[(1-\rho)\xi(1-\sigma_2) + \lambda\eta] \qquad \text{(C8)}$$

$$\widehat{U} = \frac{[(1-\sigma_1)\pi_1]^{-\beta}\pi_2}{1-\beta}[\rho\lambda(1-\sigma_1) + (1-\rho)\xi(1-\sigma_2)]. \qquad \text{(C9)}$$

Note also that we can rewrite (A7) as

$$\rho\lambda(1-\sigma_1) + (1-\rho)\xi(1-\sigma_2)$$
$$= \frac{(1-s)(1-\beta)}{s(1-\alpha)}[\rho\lambda\sigma_1 + (1-\rho)\xi\sigma_2]. \qquad \text{(C10)}$$

Putting (C4)–(C9) into (C3), using (C10) and collecting terms we can write the first-order condition as

$$\phi(\lambda) = \frac{\frac{t}{s}\left[(1-\rho)\xi\sigma_2 - \lambda\eta\right] + \frac{(1-t)}{(1-s)}\left[(1-\rho)\xi(1-\rho_2) + \lambda\eta\right]}{\frac{t}{s}\left[\rho\sigma_1 + \eta\right] + \frac{(1-t)}{(1-s)}\left[\rho(1-\sigma_1) - \eta\right]}$$

or

$$\phi(\lambda) = \frac{(1-\rho)\xi\left[\frac{t}{s}\sigma_2 + \left(\frac{(1-t)}{(1-s)}\right)(1-\rho_2)\right] - \lambda\eta\left[\frac{t}{s} - \frac{(1-t)}{(1-s)}\right]}{\rho\left[\frac{t}{s}\sigma_1 + \left(\frac{(1-t)}{(1-s)}\right)(1-\rho_1)\right] + \eta\left[\frac{t}{s} - \frac{(1-t)}{(1-s)}\right]}$$

which is equation (16) in the text.

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